Abstracts

Centralizers and split extensions of lattice-ordered groups

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A lattice-ordered group is a set equipped with both a group structure and a lattice structure such that the underlying order relation is invariant under translations. This can be defined as an algebraic structure of signature $\cdot, e, ^{-1}, \vee, \wedge$ satisfying the axioms of groups, the axioms of lattices, and the axioms related to the distributivity of the group product over both the lattice operations. In this talk, we will present various results concerning the concept of commutation in lattice-ordered groups. Our focus will be on providing an explicit description for centralizers of subalgebras. Moreover, we will examine how this characterization allows us to derive various algebraic properties about the variety of lattice-ordered groups. Furthermore, we will conduct an in-depth investigation into the concept of split extensions in the context of lattice-ordered groups.

Closure operators on subgroup lattices and the group $\mathrm{GL}(V)$

Luca Di Gravina
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Closure operators on posets play an important role in combinatorics. Let G be a group. In order to define a suitable closure operator on the subgroup lattice of G, it is desirable to exploit some natural permutation action of G on a specific ordered set. Then, for a finite vector space V, we consider the general linear group $\mathrm{GL}(V)$. A relation between its subgroup lattice and the subspace lattice of V is established by using a closure operator on the subgroup lattice of $\mathrm{GL}(V)$. Some questions and results arise from this relation.

Mumford representation and Riemann Roch space of a divisor on a hyperelliptic curve

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For a hyperelliptic curve H of genus g (with a Weierstrass point Ω, taken as the point at infinity), we determine a basis of the Riemann-Roch space

$L(\Delta + m\Omega)$, where Δ is of degree zero, directly from the Mumford representation of Δ. This stresses the meaning of the Mumford representation of Δ in this context. This provides in turn a generating matrix of a Goppa code, and we show a toy model of optimal code. Joint work with Giuseppe Filippone.

Classification of a family of 5-dimensional binary Lie algebras

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The five-dimensional anti-commutative algebras which are semidirect sums of the one-dimensional abelian algebra and a four-dimensional nilpotent Lie algebra and at the same time the two-dimensional non-abelian Lie algebra and the three-dimensional abelian algebra will be called $\mathcal{M}^5$-algebras. These algebras have analogous flags of subalgebras like the five-dimensional solvable Malcev algebras and can therefore be considered their closest relatives. The aim of this talk is to find normal forms for the binary Lie $\mathcal{M}^5$-algebras and determine their isomorphism classes.

Toroidal groups and non-totally real number fields

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A toroidal group is a complex Lie group $\mathbb{C}^n/\Lambda$, with Λ a lattice of $\mathbb{C}^n$, which does not admit non-constant holomorphic functions. We investigate the relationship between non-totally real number fields K and toroidal groups $\mathcal{T}$, as well as meromorphic periodic functions, utilizing a representation of $\mathcal{T}$ as the generalized Jacobian $\mathfrak{J}_L(\mathcal{C})$ of a suitable elliptic curve $\mathcal{C}$. We focus on the cubic and quartic cases and we write down the relations between the minimal polynomial of a suitable primitive element of K and the parameters defining the generalized Jacobian $\mathfrak{J}_L(\mathcal{C})$ corresponding to the toroidal group associated with the ring of integers. Additionally, for such a toroidal group we explicitly show the analytic and rational representations of its ring of endomorphisms, with the former giving in turn a new (complex) representation of the ring of integers of K. Moreover, for the cubic case, we give an explicit description of the m-torsion of $\mathcal{T}$ in the geometric correspondence of $\mathcal{T}$ with $\mathfrak{J}_L(\mathcal{C})$, as image of a fractional ideal of K. Joint work with Alessandro Dioguardi Burgio and Giovanni Falcone.

REFERENCES
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The Green-Tao theorem for number fields and beyond

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The famous Green-Tao theorem states that there are arbitrarily long arithmetic progressions of prime numbers. In 2020, we extended it to general number fields in joint work with Mimura, Munemasa, Seki and Yoshino. In this talk I would like to explain this result and my recent effort to deepen it. The latter involves analysis on groups of ideles and polynomial sequences on nilpotent Lie groups.

Isotopisms of nilpotent Leibniz algebras and Lie racks

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Leibniz algebras were introduced by J.-L. Loday in 1993 as a non-antisymmetric version of Lie algebras. Many results of Lie algebras have been extended to Leibniz algebras. One of them is the *Levi decomposition*, which states that every finite-dimensional Leibniz algebra $\mathfrak{g}$ is the semidirect product of a solvable ideal and a semisimple subalgebra. This makes clear the importance of the problem of Lie / Leibniz algebra classification, which has been dealt with since the early 20th century. However in general, given two Leibniz algebras $\mathfrak{g}$ and $\mathfrak{h}$, it is hard to check if $\mathfrak{g}$ and $\mathfrak{h}$ are isomorphic or not, but it is easier to see if there exists an *isotopism* between them, i.e. if there is a triple of linear isomorphisms $(f, g, h)\colon \mathfrak{g} \leftrightarrows \mathfrak{h}$ such that

$$[f(x), g(y)]_{\mathfrak{h}} = h([x, y]_{\mathfrak{g}}), \quad \forall x, y \in \mathfrak{g}.$$

The notion of isotopism between two algebraic structures was explicitly introduced in 1942 by Abraham Adrian Albert in order to classify non-associative algebras and we can use it in the case of Leibniz algebras.
In this talk we study the isotopism classes of two-step nilpotent Leibniz algebras. We show that every nilpotent Leibniz algebra with one-dimensional commutator ideal is isotopic to the *Heisenberg algebra* $\mathfrak{h}_{2n+1}$ or to the Heisenberg Leibniz algebra $\mathfrak{l}_{2n+1}^{J_1}$, where J_1 is the $n \times n$ Jordan block of eigenvalue 1. We also prove that two such algebras are isotopic if and only if the Lie racks integrating them are isotopic. This gives the classification, up to isotopism, of Lie racks whose tangent space at the unit element is a nilpotent Leibniz algebra with commutator ideal of dimension one. Eventually we introduce new isotopism invariants for Leibniz algebras and Lie racks. Joint work with Gianmarco La Rosa and Gábor P. Nagy.

REFERENCES

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Embeddings into finitely presented simple groups

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In 1973, William Boone and Graham Higman proved that a finitely generated group G has a solvable word problem if and only if G can be embedded into a simple subgroup of a finitely presented group. They conjectured a stronger result, namely that every such group G embeds into a finitely presented simple group. This conjecture remains open after almost 50 years, but recent advances in the study of finitely presented simple groups have made it possible to verify the Boone-Higman conjecture for several large classes of groups. In this talk, I will survey results on Boone-Higman embeddings of right-angled Artin groups, countable abelian groups, contracting self-similar groups, and hyperbolic groups. Joint work with Jim Belk, Collin Bleak, James Hyde, and Matthew Zaremsky.

Cohomology 2-groups: an invitation to higher dimensional categorical algebra

Giuseppe Metere
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2-groups are monoidal groupoids such that all their objects are weakly invertible with respect to the tensor product. They have been introduced in the seventies by the Vietnamese mathematician Hoàng Xuân Sính, a student of Alexander Grothendieck. In a suitable sense, they are a 2-dimensional version of the usual algebraic notion of a group. They appear quite naturally in the study of well-known constructions in (homological) algebra, where they can offer a more conceptual point of view. In my talk, I will give a brief survey on the subject, with a focus on the definition of the cohomology 2-group associated with a group-module ([1]). Joint work with Alan S. Cigoli and Sandra Mantovani.

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Covering groups of minimal exponent

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Presenting a finite group by a free product of finite cyclic groups the Hopf formula for the Schur multiplier affords also a covering group, and the cover has minimal exponent provided that the presentation preserves the order of the generators. This condition corresponds to a covering projection between simplicial complexes, and so a presentation by a Fuchsian group generated by elliptic elements corresponds to a covering projection between simplicial compact oriented surfaces.

The number of irreducible characters in a Brauer p-block

Lucia Sanus
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Let G be a finite group, p a prime number and B a Brauer p-block with defect group D. A classic problem in character theory is to analyze the properties of D through the knowledge of some invariants of B. Let $k(B)$ be the number of irreducible complex characters in B. We will discuss some recent results on the possible structures of D for small values of $k(B)$. Joint work with J. M. Martinez and N. Rizo.

Thompson-like groups acting on fractals

Matteo Tarocchi
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The trio of Thompson groups F, T and V has made its appearance in many different topics, and these groups are so ubiquitous that M. Brin called them “Chameleon groups” in 1996. Introduced in the ’60s by Richard Thompson, the groups T and V were the first examples of infinite finitely presented simple groups, whereas the fame of its smaller sibling F mostly originates from a question that has been open for decades and still is: whether it is amenable or not. In 2019 J. Belk and B. Forrest introduced a generalization of Thompson groups, the family of Rearrangement Groups. These are groups of certain “piecewise-canonical” homeomorphisms of fractals that act by permuting the self-similar pieces that make up the fractal. This talk will introduce Thompson groups and Rearrangement Groups, and will highlight some known facts about them.

On Quadratic Rational Groups

Marco Vergani
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In this seminar I’ll talk about families of groups that have a characterization of their integral central units inside the rational group algebra. Quadratic rational groups are finite groups that have a rational field of values for any irreducible character with degree over $\mathbb{Q}$ at most two. From a character table perspective, they are the ”dual” definition of the well studied semirational groups.