ON GROUPS WITH MANY GENERALIZED NORMAL SUBGROUPS

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When $\mathcal{P}$ is a property pertaining to subgroups of a group, a group G is said to have the *weak* minimal condition on $\mathcal{P}$-subgroups

Min-∞-$\mathcal{P}$

if there are **no** infinite descending chains of $\mathcal{P}$-subgroups of G

$$H_0 \gg H_1 \gg \ldots \gg H_n \gg \ldots$$

in which *all indices* $|H_i : H_{i+1}|$ *are infinite.*

When $\mathcal{P}$ is simply the property of being a subgroup, it was proved by Baer and by Zacev that for locally (soluble-by-finite) groups the weak minimal condition (and weak maximal condition) characterizes soluble-by-finite minimax groups, i.e., groups having a finite series whose factors either satisfy the minimal or the maximal condition on subgroups.

Many papers deal with (generalized soluble) groups satisfying some weak chain condition on non-$\mathcal{P}$ -subgroups for some relevant choice of the property $\mathcal{P}$, for example being abelian, normal, almost normal, nearly normal. Recall that a subgroup H of a group G is said to be almost normal (nearly normal, resp.) if the normalizer of H has finite index in G (the subgroup H has finite index in a normal subgroup of G)

I will discuss results for the cases when $\mathcal{P}$=(normal or abelian), when $\mathcal{P}$=core-finite i.e. a normal sugroup of G has finite index in H and when $\mathcal{P}$ is the property of being commensurabe with a normal subgroup of G. Recall that two subgroup are called commensurable if their intersection has finite index in both of them.

Parts of this talk are joint with Fausto De Mari and Silvana Rinauro.

Zerosums in a Galois field [1]

Giovanni Falcone

in collaboration with
Marco Pavone

Abstract

In this talk we consider a Galois field $\mathcal{P}$ of odd characteristic, and study the family $\mathcal{B}_k^x$ of all the k-sets of elements of $\mathcal{P}$ summing up to a given element x, together with the subfamily $\mathcal{B}_k^x(y)$ of all the k-sets in $\mathcal{B}_k^x$ that contain a given element y. The main results of the paper are the computation of the cardinalities b_k^x and $r_k^x(y)$ of $\mathcal{B}_k^x$ and $\mathcal{B}_k^x(y)$, respectively, for all possible values of k, x, and y, and the characterization, for $x = 0$, of the permutations of $\mathcal{P}$ inducing permutations of $\mathcal{B}_k^0$.

Besides its intrinsic combinatorial interest, this allows us, as a main consequence, to establish under what conditions on k and x the coefficients b_k^x and $r_k^x(y)$ represent the number of blocks and the replication number, respectively, of a 1-design or a 2-design $\mathcal{D} = (\mathcal{P}, \mathcal{B}_k^x)$, and, in the latter case, to characterize its automorphism group.

All these questions are answered also in the even harder case where the elements of the k-sets are required to be all nonzero, and, in fact, the two cases prove to be intrinsically inseparable.

[1] AMS MSC 2010: 11P70, 11B75, 05B05.

Engel elements in groups of automorphisms of rooted trees

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Abstract

The Grigorchuk group is an example of a group where the set of left Engel elements is not a subgroup. In this talk we survey recent results about the sets of (bounded) left or right Engel elements in some general families of groups of automorphisms of rooted trees. This is joint work with A. Garreta, M. Noce, and G. Tracey.

Topological loops having solvable indecomposable Lie groups as their multiplication groups

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Abstract

The multiplication group $Mult(L)$ and the inner mapping group $Inn(L)$ of a loop L are important tools for the investigation of the structure of L since there are strong connections between the structure of the groups $Mult(L)$ and $Inn(L)$ and that of L. R. H. Bruck proved that if the group $Mult(L)$ is nilpotent, then the loop L is centrally nilpotent and the group $Inn(L)$ is abelian. A. Vesanen showed that if the loop L is finite and the group $Mult(L)$ is solvable, then L is classically solvable, this means there exists a series of subloops of L of the form $\{e\} = L_0 \leq L_1 \leq \cdots \leq L_n = L$ such that L_{i-1} is a normal subloop in L_i and L_i/L_{i-1} is an abelian group for all $i = 1, \ldots, n$.

We prove that the solvability of the multiplication group $Mult(L)$ of a topological loop L of dimension three forces that L has a normal series whose factor loops are 1-dimensional abelian groups. The loop L is congruence solvable if and only if either L has a non-discrete centre or L is an abelian extension of a normal subgroup $\mathbb{R}$ by the 2-dimensional non-abelian Lie group or by an elementary filiform loop. In particular, if the solvable indecomposable Lie group $Mult(L)$ has dimension ≤ 6, then the loop L is centrally nilpotent of class 2.

Majorana representations of the symmetric groups

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Abstract

Majorana representations of finite groups have been introduced by A.A. Ivanov as a tool to better understand the Monster and its representation on the Conway-Norton-Griess algebra.

In my talk I shall give a motivation for the study of Majorana representations of the symmetric groups, present the state of the research and outline the combinatorial methods used.

A reduction Theorem for nonsolvable finite groups

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Abstract

Every finite group G has a normal series each of whose factors is either a solvable group or a direct product of nonabelian simple groups. The minimum number of nonsolvable factors attained on all possible such series is called the nonsolvable length of the group and denoted by $\lambda(G)$. For every integer n, we define a particular class of groups of nonsolvable length n, called n-rarefied, and we show that every finite group of nonsolvable length n contains an n-rarefied subgroup. As applications of this result, we improve the known upper bounds on $\lambda(G)$ and determine the maximum possible nonsolvable length for permutation groups and linear groups of fixed degree respectively dimension.

Universitá Statale di Milano

Character correspondences for symmetric and complex reflection groups

Eugenio Giannelli

In 2016 Ayyer, Prasad and Spallone proved that the restriction to S_{n-1} of any odd degree irreducible character of the symmetric group S_n has a unique irreducible constituent of odd degree. This result was later generalized by Isaacs, Navarro, Olsson and Tiep. In this talk I will survey some recent developments on this topic.

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THE ADDITION THEOREM FOR THE ALGEBRAIC ENTROPY

ANNA GIORDANO BRUNO

Let G be a locally compact group, $\phi : G \to G$ a continuous endomorphism and N a closed normal ϕ-invariant subgroup of G. We say that the addition theorem holds for the algebraic entropy h_{alg} if

$$h_{alg}(\phi) = h_{alg}(\phi \restriction_N) + h_{alg}(\overline{\phi}),$$

where $\overline{\phi} : G/N \to G/N$ is induced by ϕ.

It is known that the addition theorem holds when G is a discrete abelian group and that it does not hold in general. Nevertheless, we prove that the addition theorem holds when G is a discrete locally finite and normal group (i.e., G is a torsion FC-group), and when G is strongly compactly covered and ϕ is a topological automorphism.

On purely (non-) strongly real Beauville p-groups

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Abstract

A finite group G is called a *Beauville group* if it is a 2-generator group and there exists a pair of generating sets $\{x_1, y_1\}$ and $\{x_2, y_2\}$ of G such that $\Sigma(x_1, y_1) \cap \Sigma(x_2, y_2) = 1$, where $\Sigma(x_i, y_i)$ denotes the union of the subgroups that belong to the conjugacy classes of $\langle x_i \rangle$, $\langle y_i \rangle$ and $\langle x_i y_i \rangle$ for $i = 1, 2$. Then $\{x_1, y_1\}$ and $\{x_2, y_2\}$ are said to form a *Beauville structure* for G. If we further have an automorphism $\theta \in \mathrm{Aut}(G)$ and elements $g_i \in G$ for $i = 1, 2$ such that

$$g_i \theta(x_i) g_i^{-1} = x_i^{-1} \quad \text{and} \quad g_i \theta(y_i) g_i^{-1} = y_i^{-1},$$

then the Beauville structure $\{\{x_1, y_1\}, \{x_2, y_2\}\}$ is called *strongly real*.

Beauville groups typically have numerous Beauville structures some of which are strongly real, some of which are not. We are interested in the extreme cases: a Beauville group G is *purely strongly real* if every Beauville structure for G is strongly real. If none of the Beauville structures for G are strongly real then G is called *purely non-strongly real*.

In this talk, we will study Beauville p-groups which fall into one of the extreme cases mentioned above.

Affine Steiner loops

Carolin Hannusch

April 8, 2019

Joint work with Giovanni Falcone (Palermo) and Ágota Figula (Debrecen)

A *Steiner triple system* is a pair $(\mathcal{S}, \mathcal{T})$, where $\mathcal{T}$ is a family of triples of elements of $\mathcal{S}$ such that any two elements of $\mathcal{S}$ are contained exactly in one triple of the family $\mathcal{T}$. As such, a Steiner triple system with n elements is a $2-(n,3,1)$ balanced incomplete block design.
A *loop* is a set L of elements endowed with a binary operation $(\circ)$ which does not need to be associative, which has a neutral element Ω, and which is such that the equations $a \circ x = b$ and $y \circ a = b$ determine unique solutions $x = a \backslash b$ and $y = b/a$.
The affine Steiner loop associated to the STS $(\mathcal{S}, \mathcal{T})$ is defined as follows:

Definition 1. *Let $\mathcal{S}$ be a Steiner triple system, and let $\Omega \in \mathcal{S}$ be fixed. For each $x \in \mathcal{S}$, define its opposite $-x$ as the third point $\mu(x)$ in the triple $\{x, \Omega, \mu(x)\}$ through x and Ω, define the addition $x + \Omega = x$, $x + x = -x$, and, for any $y \neq x \in \mathcal{S} \setminus \{\Omega\}$,*

$$x + y = -z,$$

where z is the third point in the triple through x, y.

Affine Steiner loops behave to elementary abelian 3-groups as Steiner Triple Systems behave to affine geometries over $GF(3)$. In this talk, several properties of these loops and their multiplication group will be given, often in connection to finite geometries. One of our results is that if the loop of an $STS(n)$ is simple, then its multiplication group is the alternating group A_n.

Freely Indecomposable Groups

Wolfgang Herfort - University of Technology, Vienna, Austria

Abstract

It is proved that groups with a certain property, termed by us $\mathcal{U}$-Higman complete, cannot split nontrivially as a free product. From this somewhat technical result a number of facts like free indecomposability of inverse limits of groups and non discrete locally compact groups follow quickly. Using our device, another proof of a result of K. Eda from 1992 concerning mapping an inverse limit of groups to the *topologists product* of groups is given. Joint work with Wolfram Hojka.

On the second maximal groups
with respect to the sum of element orders

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Let G be a finite group and consider the function ψ defined by

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element $x \in G$.

In [1] H. Amiri, S.M. Jafarian Amiri and M. Isaacs proved that if G has order n and C_n denotes the cyclic group of order n, then $\psi(G) \leq \psi(C_n)$, and $\psi(G) = \psi(C_n)$ if and only if $G \simeq C_n$.
Other results have been obtained by H. Amiri, S.M. Jafarian Amiri, M. Amiri, Y. Marefat, A. Iranmanesh, A. Tehranian, R. Shen, G. Chen and C. Wu.

We will report some new results concerning the function ψ, jointly obtained with Marcel Herzog and Mercede Maj in the papers [2], [3], [4], [5]. In particular we will present some better upper bounds for $\psi(G)$ when G is not cyclic, and some results on the structure of G assuming some bounds on $\psi(G)$.

References

[1] H. Amiri, S.M. Jafarian Amiri, I.M. Isaacs, Sums of element orders in finite groups, *Comm. Algebra* **37** (2009), 2978-2980.

[2] M. Herzog, P. Longobardi, M. Maj, An exact upper bound for sums of element orders in non-cyclic finite groups, *J. Pure Appl. Algebra*, **222** n.7 (2018), 1628-1642

[3] M. Herzog, P. Longobardi, M. Maj, Sums of element orders in groups of order $2m$ with m odd, *Comm. Algebra*, doi.org/10.1080/00927872.2018.1527924.

[4] M. Herzog, P. Longobardi, M. Maj, Two criteria for solvability of finite groups, *J. Algebra*, **511** (2018), 215-226.

[5] M. Herzog, P. Longobardi, M. Maj, The second maximal groups with respect to the sum of element orders, *submitted*, arxiv.org/abs/1901.09662.

The Theorem of Norton-Sakuma

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Abstract

A central role for understanding the representations of the Monster and most of its subgroups on the 196884-dimensional Conway-Norton-Griess algebra is played by Norton's classification of its subalgebras that are generated by two axes. Norton's result was remarkably extended by Sakuma to the context of certain conformal-field-theory-type vertex operator algebras related to Majorana fermions and it was, subsequently, obtained in full generality within the axiomatics of axial algebras by Ivanov, Seress, Pasechnik, Shpectorov, Hall, Rehren, Franchi and Mainardis. In my talk I shall present this result and its relations to the theory of Majorana representations.

Rational Embeddings of Hyperbolic Groups

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Abstract

For a finitely generated group, the Cayley graph is a metric space encoding the structure of the group. Gromov introduced the notion of a δ-hyperbolic group, a finitely generated group with a negatively curved Cayley graph, that is, for any triangle in the graph with geodesic sides, each side is contained in the δ-neighborhood of the union of the two other sides. Hyperbolic groups are "prevalent" among finitely generated groups.

Grigorchuk, Nekrashevych and Sushchanskii defined the rational group as the full group of homeomorphisms of a Cantor space and which admit precisely finitely many types of "local actions" described by finite state transducers (one of many models of computing machines). This is a rather large group and, by construction, it contains all groups generated by finite state automata (for example, the Grigorchuk group of intermediate word growth).

In this talk I will introduce these groups and some of their properties and explain how to embed a class of hyperbolic groups in the rational group.

Parts of this talk are joint with James Belk, Collin Bleak and James Hyde.

On the classification of Schreier extensions of monoids with non-abelian kernel

Andrea Montoli

Groups and topological groups, Milano, June 21-22, 2019

Abstract

We show that any regular (right) Schreier extension of a monoid M by a monoid A induces an abstract kernel $\Phi\colon M \to \frac{End(A)}{Inn(A)}$. If an abstract kernel factors through $\frac{SEnd(A)}{Inn(A)}$, where $SEnd(A)$ is the monoid of surjective endomorphisms of A, then we associate to it an obstruction, which is an element of the third cohomology group of M with coefficients in the abelian group $U(Z(A))$ of invertible elements of the center $Z(A)$ of A, on which M acts via Φ. An abstract kernel $\Phi\colon M \to \frac{SEnd(A)}{Inn(A)}$ is induced by a regular weakly homogeneous Schreier extension of M by A if and only if its obstruction is zero. We also show that the set $Ext(M, A, \Phi)$ of isomorphic classes of regular weakly homogeneous Schreier extensions inducing a given abstract kernel $\Phi\colon M \to \frac{SEnd(A)}{Inn(A)}$, when it is not empty, is in bijection with the second cohomology group of M with coefficients in $U(Z(A))$. This is joint work with Nelson Martins-Ferreira, Alex Patchkoria and Manuela Sobral.

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Víctor Manuel Ortiz-Sotomayor (Universitat Politècnica de València, Spain): "Conjugacy classes of vanishing elements contained in a normal subgroup"

Abstract: Within finite group theory, the relationship between the structure of a group and certain data extracted from its character table has been widely investigated. In particular, the columns that correspond to zeros of the irreducible complex characters have attracted interest (see for instance the recent survey [1]). Those conjugacy classes of a group G are the so-called *vanishing* conjugacy classes of G. The aim of this talk is to show how the normal structure of G is affected by the vanishing conjugacy classes of G that lie in a normal subgroup.

Joint work with María José Felipe and Nicola Grittini ([2]).

References

[1] S. Dolfi, E. Pacifici and L. Sanus: *On zeros of characters of finite groups.* Springer Nature Singapore, Group Theory and Computation, Indian Statistical Institute Series. 41-58 (2018).

[2] M.J. Felipe, N. Grittini and V.M. Ortiz-Sotomayor: On zeros of irreducible characters lying in a normal subgroup. *Submitted* (2019).

Graphs on class sizes and character degrees

Lucía Sanus

Abstract

Let X be a set of positive integers. The prime graph for X, denoted by $\Delta(X)$, is the (simple undirected) graph whose vertices are the prime divisors of the numbers in X, and two distinct vertices p, q are adjacent if and only if pq divides some number in X. We recall that the complement graph $\overline{\Delta}(X)$ of $\Delta(X)$ is defined as the graph having the same vertex set, in which two vertices are adjacent if and only if they are not adjacent in $\Delta(X)$. We present recent results on these two graphs being X either the conjugacy class sizes or the character degrees of a finite group G.

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Characters of degree not divisible by two primes

Carolina Vallejo Rodríguez

For a set of primes π, let $\mathrm{Irr}_{\pi'}(G)$ be the set of irreducible characters of G of degree coprime to p, for every p in π. Also, denote by $\mathrm{Lin}(G)$ the set of linear characters of G. In the case where π consists of a single prime p, a celebrated theorem of J. Thompson asserts that, if $\mathrm{Irr}_{\pi'}(G) = \mathrm{Lin}(G)$ then G has a normal p-complement. In this talk I will mainly discuss the case where π contains two primes.
This is joint work with Eugenio Giannelli and Mandi Schaeffer Fry.

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