International Conference on Topological Algebras and Their Applications

Palermo, August 31 - September 2, 2022

The Organizers:

M. S. Adamo, G. Bellomonte, G. Falcone, C. Trapani, S. Triolo, F. Tschinke

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Abstracts

Mart Abel
Tallinn University/University of Tartu

About the joint continuity of the module multiplication

Let (A, tau_A) be any topological algebra over $\mathbb{R}$ or $\mathbb{C}$ and (E, τ_E) a topological left, right or two-sided A-module with jointly continuous module multiplication. We introduce the results of our recent study about the "passing on" the joint continuity of module multiplication to different new structures, constructed form one or more topological left (right or two-sided) A-modules (like submodules, quotient modules, direct sums, projective limits, injective limits, tensor products, etc.). The content of the talk generalizes the results of the paper [1] and is based on the submitted paper with the same title as the talk.

[1] Haralampidou, M., Oudadess, M, Palacios, L., Signoret, C., *On locally A-convex modules*, Mediterr. J. Math. **19**, 23 (2022).

Mati Abel
University of Tartu

Waelbroeck (not necessarily convex) algebras

The concept of Waelbroeck algebra has been introduced by L. Waelbroeck in 1954 under the name ,,continuous inverse algebra" for locally convex algebras. The name "Waelbroeck algebra" was first used by R. Ouzilou in 1965 (for locally convex case) and by A. Mallios in 1986 (for arbitrary case). So far, the main properties of locally convex Waelbroeck algebra are known. Various properties of Waelbroeck algebra for general topological algebras are presented in this talk. The concept of one-sided Waelbroeck algebra is introduced and some problems for such topological algebras are given.

Key words; Waelbrock algebra , one-sided Waelbroek algebra, locally convex algebra, topological algebra.

Hugo Arizmendi Peimbert
and Ángel Carrillo Hoyo
University Autónoma de México

On the nn-bounded linear operators on a locally convex space

Let X be a locally convex space. Let T be a linear operator on X. We say that T is *nn-bounded* if there exists a base of neighborhoods of zero in X such that T maps every neighborhood in this base into a multiple of itself. In terms of seminorms. If there exists a generating family of seminorms P such that for every p in P there is a positive scalar a satisfying $p(T(x)) < a\,p(x)$. This defines a family of associated seminorms.

I should remark that this definition depends upon the basis, as we show by one example.

We denote by $Nn(P)$ the collection of all nn-bounded operators, which is a locally convex algebra under the usual linear operations, the composition as multiplication and endowed with the topology given the family of the associated seminorms.

In this talk we give necessary conditions that imply that a linear operator on X be a nn-bounded operator. We endow to this topological algebra $Nn(p)$ with the topology of the *nn-convergence.*

We define the *radius of nn-boundeness* of a bounded operator T and we show several properties of this radius.

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joint work with Camillo Trapani
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*Order bounded elements in locally convex quasi *-algebras*

A new notion of order bounded element in a locally convex quasi *-algebra is defined and studied, attempting to obtain results similar to those already known in the Banach case. The notion involves a family of invariant positive sesquilinear forms which allow a closed cyclic *-representation of a locally convex quasi *-algebra with unit.

Jean Esterle
University of Bordeaux

Picard-Borel ideals in Fréchet algebras are prime

A Picard-Borel algebra is a commutative unital complex algebra A such that every family of pairwise linearly independent invertible elements of A is linearly independent, and a Picard-Borel ideal I in a commutative complex unital algebra A is an ideal I of A such that the quotient algebra A/I is a Picard-Borel algebra. A celebrated theorem of Borel going back to 1897 shows that the algebra $\mathcal{H}(\mathbb{C})$ of entire functions is a Picard-Borel algebra

There are easy examples of Picard-Borel algebras having zero divisors such that $\mathbb{C}.1 \subsetneq Inv(A)$. The main result of the talk is that all Picard-Borel ideals in commutative unital Fréchet algebras are prime. This result seems to be relevant for Michael's problem, since dense Picard-Borel ideals could play a role in the construction of discontinuous characters on some commutative unital Fréchet algebras, as suggested by the author in a work which was initiated at Winnipeg during the 1993 Banach Algebras conference. For closed Picard-Borel ideals a proof of this result was given at ICTAA 2021 in Tartu.

Giovanni Falcone

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What is the image of a 2-variate meromorphic function, with three real-independent periods?

We describe the 2-variate meromorphic function $G(z_1, z_2)$ having three given real-independent periods. It is harmless to assume that the periods are $\{(1,0),(0,1),(\hat{\tau},\tilde{\tau})\} \subset \mathbb{C}^2$, thus we find

$$G(z_1, z_2) = \exp(-2\eta_1 \widetilde{\tau} z_1) \frac{\sigma(z_M)}{\sigma(z_N)} \frac{\sigma(z_1 - z_N)}{\sigma(z_1 - z_M)} \ \exp(2\pi\, i z_2)$$

where σ is the Weierstrass sigma function with fundamental parallelogram generated by $\{1, \widehat{\tau}\}$, η_1 is a constant depending on σ, and v_M, v_N are two fixed complex numbers, the role of which will be described in detail.

This gives us the occasion of giving a (very short) recap of an exciting period in the History of mathematics, as well as a (very short) description of the connections between Geometry, Algebra and Complex analysis.

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Some new results on GB-algebras*

The talk **is devoted to the fond memory of** the late Professor
MOHAMED OUDADESS

In this talk we shall first discuss an extension of the Ogassawara's commutativity condition for the so called GB*-algebras (Allan). The result is due to M. Oudadess, which reads as follows: *if A is a GB*-algebra and x, y are any positive elements in A, then A is commutative, if and only if, $x \leq y$ always implies $x^2 \leq y^2$.*

A GB*-algebra is an unbounded generalization of a C*-algebra and the same is true for an AO*-algebra (Lassner). The Arens algebra $L^\omega[0,1]$ is an example of a GB*-algebra and of an AO*-algebra. A second result that we shall discuss is joint with A. Inoue, M. Weigt and I. Zarakas; it connects GB*-algebras with AO*-algebras and it is a partial extension of a more general result of K. Schmüdgen in our case. Namely, we have proved that *a Frèchet GB*-algebra A is an AO*-algebra, if and only if, for every self-adjoint continuous linear functional f on A there exist two continuous, positive, linear functionals on A, whose difference is f.*

Alexander Ya. Helemskii
jointly with T.Oikhberg
Moscow State University

Free and projective multinormed spaces and their generalizations

p-Multinormed spaces, $p \in [1, \infty]$), present a comparatively new structure of functional analysis. It gradually appeared in papers of quite a few mathematicians. These spaces found important applications in the theory of Banach lattices and some other areas. Also they present an independent interest as a certain "lighter" version of quantized normed spaces.

In this talk we give a full description of projective (= homologically best) p-multinormed spaces. The result is obtained as a partial case of a result concerning the so-called **L**-spaces. These are far-reaching generalizations of p-multinormed spaces: we get the latter spaces when $\mathbf{L} := L_p(X)$, and the measure space X is $\mathbb{N}$ with the counting measure.

As a main tool, we use the general-categorical concept of freeness. We introduce and characterize free **L**-spaces, defined in terms on some naturally appearing functor. After this, using general-categorical applications of freeness to projectivity, we obtain a broad class of projective **L**-spaces. In "nice" cases (e.g., $\mathbf{L} := L_p(X)$) the latter happen to be, informally speaking, "well situated" complemented subspaces of free spaces.

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An Unbounded Generalization of Tomita's Observable Algebras

In quantum mechnics, self-adjoint operators A in a Hilbert space $\mathcal{H}$ with inner product $(\cdot|\cdot)$ represent observables describing a given quantum system, while unit vectors x in $\mathcal{H}$ represent states of the system. The value $(Ax|x)$ for any state x is the expectation of the observable A in the state x. The case that the correspondence of an operator observable A in a quantum system to a state x is a mapping, namely A determines the unique state x, has been considered often. If nothing like that happens in a physical phenomenon how then can one approach this phenomenon mathematically? Tomita's idea for this question is to consider the quadruplet (A, x, y^*, μ) consisting of an observable A, of two states x, y and of an expectation μ as an observable, which is called a quadruplet observable, and to introduce an algebraic and topological structure in the set of quadruplet observables. This means that an operator observable A having two different states can be regarded as two different observables. One of the authors has introduced this theory in [2]. The operator part of a Tomita's observable is always a bounded linear operator on $\mathcal{H}$, however an operator observable in quantum mechanics is unbounded, and the GNS-representation of a positive linear functional on a $*$-algebra is unbounded in general. This is our motivation for defining and studying unbounded observable algebras, and for applying them to studies of positive linear functionals on $*$-algebras. Here we state this roughly. Let $\mathcal{D}$ be a dense subspace in a Hilbert space $\mathcal{H}$ and write $\mathcal{D}^* = \{\xi^* \in \mathcal{H}^*; \xi \in \mathcal{D}\}$, where ξ^* is an element of the dual space $\mathcal{H}^*$ of $\mathcal{H}$ defined by $< \xi^*, x >:= (x|\xi)$ for all $x \in \mathcal{H}$. We denote by $\mathcal{L}^\dagger(\mathcal{D})$ the set of all linear operators X from $\mathcal{D}$ to $\mathcal{D}$ satisfying $D(X^*) \supset \mathcal{D}$ and $X^*\mathcal{D} \subset \mathcal{D}$, where X^* is the adjoint of X and $D(X^*)$ is the domain of X^*. Then $\mathcal{L}^\dagger(\mathcal{D})$ is a $*$-algebra consisting of closable operators in $\mathcal{H}$ equipped with the usual operations ($X + Y, \alpha X$ and XY) and the involution $X \mapsto X^\dagger := X^* \lceil_{\mathcal{D}}$ (the restriction of X^* to $\mathcal{D}$). A quadruplet $A = (A_0, \xi, \eta^*, \mu)$ of $A_0 \in \mathcal{L}^\dagger(\mathcal{D})$, $\xi, \eta \in \mathcal{D}$ and $\mu \in \mathbb{C}$ is called an (unbounded) quadruplet observable on $\mathcal{D}$ and denoted by $Q^\dagger(\mathcal{D})$ of all quadruplet observables on $\mathcal{D}$. Refering observables, states and expectations in the standard Hilbert space formulation of the quantum mechanics, we define the algebraic operations and involution † on $Q^\dagger(\mathcal{D})$ as follows:

$$\begin{aligned} A+B &= (A_0+B_0, \xi+\zeta, \eta^*+\chi^*, \gamma+\sigma), \\ \alpha A &= (\alpha A_0, \alpha\xi, \alpha\eta^*, \alpha\gamma) \\ AB &= (A_0B_0, A_0\zeta, (B_0^\dagger\eta)^*, (\zeta|\eta)), \\ A^\dagger &= (A_0^\dagger, \eta, \xi^*, \bar{\gamma}) \end{aligned}$$

for $A=(A_0,\xi,\eta^*,\gamma)$, $B=(B_0,\zeta,\chi^*,\sigma)\in Q^\dagger(\mathcal{D})$ and $\alpha\in\mathbb{C}$. A *-subalgebra of the *-algebra $Q^\dagger(\mathcal{D})$ is said to be a $Q^\dagger$-algebra on $\mathcal{D}$. Investigating $Q^\dagger$-algebras, we may deal with various physical phenomenons. For an element $A=(A_0,\xi,\eta^*,\gamma)$ of a $Q^\dagger$-algebra $\mathfrak{A}$ on $\mathcal{D}$ we write $\pi(A)=A_0$, $\lambda(A)=\xi$, $\lambda^*(A)=\eta^*$ and $\mu(A)=\gamma$. Then π is a (possibly unbounded) $*$-representation of $\mathfrak{A}$ on $\mathcal{D}$ (namely, a $*$-homomorphism of $\mathfrak{A}$ into $\mathcal{L}^\dagger(\mathcal{D})$), λ is a vector representation of $\mathfrak{A}$ into $\mathcal{D}$ (namely, a linear mapping of $\mathfrak{A}$ into $\mathcal{D}$ satisfying $\lambda(AB)=\pi(A)\lambda(B)$ for all $A,B\in\mathfrak{A}$)), λ^* is a vector representation of $\mathfrak{A}$ into $\mathcal{D}^*$ and μ is a positive linear functional on $\mathfrak{A}$. A trio $(\pi(A),\lambda(A),\lambda^*(A))$ obtained by cutting a quadruplet observable A on $\mathcal{D}$ is called a trio observable on $\mathcal{D}$, and the set $T^\dagger(\mathcal{D})$ of all trio observables on $\mathcal{D}$ is also a $*$-algebra without identity under operations and the involution † as those in the case of $Q^\dagger(\mathcal{D})$. A $*$-subalgebra of $T^\dagger(\mathcal{D})$ is called a $T^\dagger$-algebra on $\mathcal{D}$. When $\mathcal{D}=\mathcal{H}$, a quadruplet (resp. trio) observable A is a Tomita's quadruplet (resp. trio) observable, namely, $\pi(A)$ is a bounded linear operator on $\mathcal{H}$, $\lambda(A)\in\mathcal{H}$ and $\lambda^*(A)\in\mathcal{H}^*$. The set $Q^*(\mathcal{H})$ (resp. $T^*(\mathcal{H})$) of all Tomita's quadruplet (resp. trio) observables on $\mathcal{H}$ is a Banach $*$-algebra without identity equipped with the above operations $A+B$, αA, AB, the involution $A^\sharp := (\pi(A)^*, \lambda^*(A)^*, \lambda(A)^*, \overline{\mu(A)})$ (resp. $A^\sharp := (\pi(A)^*, \lambda^*(A)^*, \lambda(A)^*)$) and the norm $\|A\| := \max(\|\pi(A)\|, \|\lambda(A)\|, \|\lambda^*(A)\|, |\mu(A)|)$ (resp. $\|A\| := \max(\|\pi(A)\|, \|\lambda(A)\|, \|\lambda^*(A)\|)$).

A $*$-subalgebra of $Q^*(\mathcal{H})$ (resp. $T^*(\mathcal{H})$) is called a Q^*-algebra (resp. a T^*-algebra) on $\mathcal{H}$, and a closed $*$-subalgebra of the Banach $*$-algebra $Q^*(\mathcal{H})$ (resp. $T^*(\mathcal{H})$) is called a CQ^*-algebra (resp. CT^*-algebra) on $\mathcal{H}$. Here we remark that in [2] the involution on $Q^*(\mathcal{H})$ and $T^*(\mathcal{H})$ is denoted by $A\to A^\sharp$ as we did, however we denote the involution on $Q^\dagger(\mathcal{D})$ and $T^\dagger(\mathcal{D})$ by $A\to A^\dagger$. The theory of unbounded observable algebras is closely related to operator algebras on Krein spaces [5] and also to unbounded Tomita-Takesaki theory [1], and is applicable to studies about invariant positive sesquilinear forms on $*$-algebras without identity.[3] We shall focus this talk on the following result:

Theorem Let $\mathfrak{A}$ be a (π, λ)-self-adjoint $T^\dagger$-algebra on $\mathcal{D}$ in $\mathcal{H}$. Then $\mathfrak{A}$ is semifinite, namely $\pi(\mathfrak{A}^c)$ is nondegenerate if and only if there exists a subset $\{\xi_\alpha\}$ of self-adjoint element of $\mathcal{D}$ satisfying

1. for $\alpha \neq \beta$, $(\pi(A)\xi_\alpha|\xi_\beta) = 0$ for all $A \in \mathfrak{A}$,
2. $\sum_\alpha \|\pi(A)\xi_\alpha\|^2 < \infty$ for all $A \in \mathfrak{A}$,
3. $\lambda(A) = \sum_\alpha \pi(A)\xi_\alpha$ for all $A \in \mathfrak{A}$.

Here we say that

- $\mathfrak{A}$ is called (π, λ)-self-adjoint if the O^*-algebra $\pi(\mathfrak{A})$ is self-adjoint and the invariant subspace $\lambda(\mathfrak{A})$ of $\mathcal{D}$ is essentially self-adjoint, namely the restriction $\pi(\mathfrak{A})\lceil_{\lambda(\mathfrak{A})}$ is essentially self-adjoint,
- $\mathfrak{A}^c$ is a commutant of $\mathfrak{A}$ defined by $\mathfrak{A}^c = \{K = (pi(K), \lambda(K), \lambda(K^\sharp)^*) \in T^*(\mathcal{H}); AK = KA, ^\forall A \in \mathfrak{A}\}$,
- an element ξ of $\mathcal{D}$ is self-adjoint if the O^*-algebra $\pi(\mathfrak{A}\lceil_{\pi(\mathfrak{A})\xi}$ is essentially self-adjoint.

Keywords: Tomita's observable algebras, O^*-algebras, $T^\dagger$-algebras.

[1] A. Inoue, *Tomita-Takesaki Theory in Algebras of Unbounded Operators*, LNM 1699, Springer-Verlag, Berlin, 1988.

[2] A. Inoue, *Tomita's Lectures on Observable Algebras in Hilbert Space*, LNM 2285, Springer-Verlag, 2021.

[3] A. Inoue and H. Inoue, *An Unbounded Generalization of Tomita's Observable Algebras.* Rep. Math. Phys., **89** (2022), 153–184.

[4] H. Inoue, *An Unbounded Generalization of Tomita's Observable Algebras II.* in preparation.

[5] Y. Nakagami and M. Tomita, *Triangular matrix representation for self-adjoint operators in Krein spaces.* Japanese J. Math. **14** (1988), 165–202.

Stefan Ivkovic

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Hypercyclic generalized shift operators over Segal algebras

We characterize hypercyclic generalized bilateral shift operators on the space of sequences whose coefficients are functions belonging to various Segal algebras. Moreover, we obtain some other properties regarding the approximate units of these algebras.

My talk is based on the following paper: [2205.05485] Hypercyclic shift operators over Segal algebras (arxiv.org)

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On mid p-summing algebras

Let $1 \leq p < \infty$. Given a Banach space $(X, \|\cdot\|)$, recall the Banach spaces $\ell_p^{strong}(X)$ and $\ell_p^{weak}(X)$ of, respectively, absolutely and weakly p-summable X-valued sequences equipped with their usual norms. We define a Banach algebra X to be a complex Banach space X equipped with a continuous multiplication

$$X \times X \longrightarrow X : (x, y) \mapsto xy$$

which is associative with respect to both vectors and scalars and distributive. Hence there exists a constant $C > 0$ such that

$$\|xy\| \leq C\|x\|\|y\|, \quad \forall x, y \in X. \tag{2.1}$$

No matter how we choose $1 \leq p \leq \infty$ and finitely many vector $x_1, \ldots, x_n$ and $y_1, \ldots y_n$ from X, it holds that

$$\|\sum_{k\leq n} x_k y_k\| \leq C\|(x_k)\|_{p^*}^{strong} \cdot \|(y_k)\|_p^{strong} \tag{2.2}$$

with C as in (2.1). If X is a Banach space, consider the vector space

$$\ell_p^{mid}(X) := \{(x_j)_{j=1}^\infty \in \ell_p^{weak}(X) | ((x_n^*(x_j))_{j=1}^\infty)_{n=1}^\infty \in \ell_p(\ell_p) \text{ whenever } (x_n^*)_{n=1}^\infty \in \ell_p^{weak}(X^*)\}$$

under pointwise operations, equipped with the norm

$$\|(x_j)_{j=1}^\infty\|_{mid,p} := \sup_{(x_n^*)_{n=1}^\infty \in B_{\ell_p^{weak}(X^*)}} \Big\{ \Big(\sum_{n=1}^\infty \sum_{j=1}^\infty |x_n^*(x_j)|^p \Big)^{1/p} \Big\},$$

under which it is a Banach space and a Banach algebra under the pointwise definitions as soon as X is a Banach algebra. Here X^* denotes the topological dual of X and B_X denotes the closed unit ball of X.

We say that the Banach algebra X is a *mid p-summing* algebra ($1 \leq p < \infty$) if there is a constant $c > 0$ (not necessarily C) such that

$$\|\sum_{k\leq n} x_k y_k\| \leq c\|(x_k)\|_{p^*}^{strong} \cdot \|(y_k)\|_{mid,p} \tag{2.3}$$

regardless of the choice of finite collections of vector $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ from X. In this paper, we undertake a systematic study of these algebras.

Lourdes Palacios
University Autónoma Metropolitana Iztapalapa

More on locally A-convex modules

Let $\mathcal{A}$ be a locally convex algebra. Several notions of A-convexity (absorbing convexity) for locally convex left $\mathcal{A}$-modules have been recently introduced. In this talk we examine some stability properties of the class of such modules. These properties are related to submodules, quotient modules, unitization, completion, direct products, finite direct sums and strict inductive limits. We also discuss an Arens-Michael-like decomposition for these modules.

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About the automatic continuity of homomorphisms and n-homomorphisms between topological algebras

Several authors have studied the automatic continuity of homomorphisms and n-homomorphisms between Banach algebras and locally m-convex algebras. I will give an overview of some results and talk about some generalizations for the automatic continuity of homomorphisms and n-homomorphisms between locally m-pseudoconvex algebras.

Pavel Ramos-Martinez
Co-authors: Lourdes Palacios, Carlos Signoret
University Autónoma de México

About the density of $CV_0(X) \otimes A$ in the space $CV_0(X, A)$.

Let X be a completely regular Hausdorff space and V a Nachbin family on X. For $(A, \{\| \|_\alpha\}_{a \in I})$ a locally convex space, let $CV_0(X, A)$ the locally convex space of all vector-valued continuous functions f such that $v(\| \|_\alpha \circ f)$ vanish at infinity for very $\alpha \in I$ and every $v \in V$, endow $CV_0(X, A)$ with the topology given by the uniform seminorms induced by every $\alpha \in I$ and every $v \in V$. Many authors have study the algebraic and topological properties of $CV_0(X, A)$, and one question in this direction is: Under what sufficient conditions on X, A or V we obtain that $CV_0(X) \otimes A$ is a dense subspace of $CV_0(X, A)$? In this talk we present some sufficient conditions on X, A or V that answers this questions, the results are based on some well known characterizations of $CV_0(X, \mathbb{C})$ as a topological algebra.

Carlos Signoret
University Autónoma Metropolitana

On tensor products and projective limits

Let X be a completely regular Hausdorff space, V a Nachbin family on X and A a locally convex algebra. For A a locally convex algebra, let $CV_0(X, A)$ be the algebra of all weighted vector-valued continuous functions and $CV_0(X)$ the algebra of all weighted scalar continuous functions, both equipped with the topology given by the uniform seminorms induced by V. Consider $(\{A_\alpha\}_{\alpha \in I}, \{\varphi_{\alpha,\beta}\}_{\alpha \preceq \beta})$ a projective system of locally convex algebras and $A = \varprojlim A_\alpha$ its projective limit algebra. In this talk we examine some relations between $CV_0(X, A)$, $CV_0(X) \otimes A$, $\varprojlim CV_0(X, A_\alpha)$ and $\varprojlim (CV_0(X) \otimes A_\alpha)$. We also discuss some conditions under which they are isomorphic. This is an ongoing research with Lourdes Palacios and Pavel Ramos-Martínez.

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*Topological quasi *-algebras, 40 years later: short survey and open problems.*

The theory of topological quasi*-algebras has developed during the last 40 years, starting from the paper of G. Lassner, appeared in 1981 [1], whose main motivation came from certain problems arising in the study of some quantum statistical models. From a mathematical point of view, they arise in natural way when completing a *-algebra $\mathfrak{A}_0$ with respect to a locally convex topology τ, if the left- and right multiplications are separately but not jointly continuous.

Many results on the theory are now at our disposal both from the purely theoretical mathematical side and for applications too. This family of mathematical objects includes also very familiar spaces of functions and operators, see [2].

After summarizing the basic definitions and facts, we will present some results concerning the special case of a locally convex quasi *-algebra $(\mathfrak{A}[\tau], \mathfrak{A}_0)$ whose distinguished *-algebra $\mathfrak{A}_0$ is a C*-normed algebra,with particular care to those having sufficiently many *-representations into partial *-algebras of closable operators in Hilbert space. Some open problems will be discussed.

2010 Mathematics Subject Classification. Primary 46L08; Secondary 46L89, 47L60.

[1] G. Lassner, *Topological algebras and their applications in quantum statistics.* Wiss. Z. KMU-Leipzig, Math. Naturwiss. R., **30** (1981), 572–595.

[2] M. Fragoulopoulou and C. Trapani *Locally Convex Quasi *-algebras and their Representations.* Lecture Notes in Mathematics 2257, Springer 2020.

Salvatore Triolo
Joint work with F.Burderi and C.Trapani
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Extensions of linear functionals

We discuss a general strategy which produces a special classes of extensions of a linear functional defined on a dense *-subalgebra $\mathfrak{A}_0$ of topological *-algebra $\mathfrak{A}[\tau]$. The obtained results are applied to the commutative integration theory to recover from the abstract setup the well-known extensions of Lebesgue integral and, in noncommutative integration theory, for introducing a generalized integral of measurable operators.

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Applications of topological algebras to irreversible quantum dynamics

It is well known that the dynamics of reversible quantum processes are described by a group of unitary operators on a Hilbert space. If one represents the observables of a quantum system, in which the dynamics is reversible, as self-adjoint elements of a locally convex $*$-algebra, then this is equivalent to dynamics being described by a $*$-automorphism group of the algebra. Since our physical world consists mainly of observables which are unbounded operators on a Hilbert space, the locally convex $*$-algebra should consist of unbounded linear operators on a Hilbert space, and should have a lot in common with C*-algebras, such as existence of positive linear functionals and representations. Generalized B*-algebras would be a good candidate to house the observables, as they are abstract $*$-algebras of unbounded linear operators which generalize C*-algebras.

If the dynamics of quantum processes in a quantum system are irreversible, then groups of $*$-automorphisms will no longer be sufficient. Instead, one requires semigroups of completely positive linear mappings. Motivated by all of this, we present in this talk some results on semigroups of completely positive linear mappings of generalized B*-algebras. Following this, we also give a Schrodinger equation for irreversible quantum processes.

List of participants

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Mati Abel* - University of Tartu (Estonia)
Maria Stella Adamo - The University of Tokyo (Japan)
Hugo Arizmendi Peimbert* - University Autónoma de México (Mexico)
Giorgia Bellomonte* - University of Palermo (Italy)
Fabio Burderi - University of Palermo (Italy)
Alessandro Dioguardi Burgio - University of Palermo (Italy)
Rachid El Harti - University of Hassan First (Morocco)
Jean Esterle* - University of Bordeaux (France)
Giovanni Falcone* - University of Palermo (Italy)
Maria Fragoulopoulou* - University of Athens (Greece)
Alexander Helemskii* - Moscow State University (Russia)
Hiroshi Inoue* - Kyushu Sangyo University (Japan)
Stefan Ivkovic* - Mathematical Institute of the Serbian Academy of Sciences and Arts (Serbia)
Gianmarco La Rosa - University of Palermo (Italy)
Charles Maepa* - University of Pretoria (South Africa)
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(A star denotes the delegates who will give a talk)